CORRECTION SHEET FOR "EXAMPLES OF COMMUTATIVE RINGS"

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Abstract. This is a list of corrections (and a few comments) to Examples of Commutative Rings, published in 1981 by Polygonal Press. I offer my apologies to all who have purchased or perused this book for the necessity of issuing this corrections sheet at all. It was, of course, my intention that the book should not contain any errors; nonetheless, it did contain some misprints and some mistakes. I hope that I have found and corrected all such errors in the present list.

I would like to offer my thanks to William Heinzer, Daniel Anderson, Hale Trotter, and Robert Gilmer for their valuable assistance in preparing this list. Heinzer, in particular, compiled a detailed list of answers to questions, misprints, and other mistakes, which has been extremely useful. If any reader should find any more errors in Examples of Commutative Rings, I would be grateful to be informed of them.

1. Misprints

p. 5 line -12.	"R(X)" should be "R[X]"
p. 6 line -10.	"oridinary" should be "ordinary"
p. 39 line 16.	"any any" should be "and any"
Ex. 2 (f) line 1.	"1972a, Exercise 21" should be "1972a, p. 321, Exercise 21"
Ex. 4 (a) line 1.	"actulally" should be "actually"
Ex. 26.	Let $x, y_1, y_2, \ldots$ be the images of $X, Y_1, Y_2, \ldots$, respectively. The indicated ideals should be described in terms of $x, y_1, y_2, \ldots$ rather than $X, Y_1, Y_2, \ldots$.
Ex. 36 (c) lines 1-2.	"VP" should be "V/P"

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Warning: this transcription of the errata to LaTeX has not yet been proofread thoroughly! Thanks to Harry C. Hutchins for providing the original errata. Please submit additions via the webform.

Ex. 67 (d) line 2.	"ans" should be "and".
Ex. 78 (e) lines 1-2.	"Example 102" should be "Example 101"
Ex. 80 line 2.	"L^p" should be "L^2"
Ex. 114 (a) line 2.	"M" should be "the preimage of M in $K[U, V, W]$".
(d) line 1.	"T is thus" should be "R is thus"
Ex. 126 (e) line 1.	"Nagata 1977" should be "Nagata 1970"
Ex. 155 (d) line 2.	"The" should be "(The"
Ex. 158 line 2.	"$v(Y) = 2$" should be "$v(Y) = \sqrt{2}$" (Or $v(Y)$ = any irrational number.)

2. Answers to Questions

p. 19 note 14.	W. Heinzer and J. Ohm have given an example of an almost Krull domain which is not pseudo-Prüfer in [4].
p. 31 lines 23-24.	J. Arnold has shown in [2] that if R is a discrete valuation domain of rank n, then $\dim(R[[X_1, X_2, \ldots, X_m]]) = nm+1$. Thus if R_0 is a discrete valuation domain of dimension 2 and $R = R_0[[X, Y]]$, then we have $\dim(R) = 5$ and $\dim(R[[Z]]) = 7$.
Ex. 48 (f) lines 1-2.	Of course, R does have finitely generated prime ideals, such as $(X_3^2 - X_1X_2)$.
Ex. 80 (g) line 3.	The completion of S is T and so is factorial.
Ex. 114 (d) lines 1-2.	R is indeed geometrically normal.
Ex. 123 (a) lines 1-2.	The rings R_m are not Noetherian and not integrally closed.
Ex. 126 (d) line 1.	R is factorial for $n \geq 5$ and may or may not be factorial for $n = 3$ and $n = 4$, depending on K. (The question is trivial for $n = 1$ and $n = 2$.)
Ex. 170 (j) lines 1-2.	T is regular. If we localize to the case in which only the two interesting maximal ideals survive, then the resulting ring will certainly be factorial. Is T Gorenstein?
Ex. 175 (e) line 1.	R is not catenary but T is Bézout.

3. Outright Mistakes

p. 4 line 6.	Irreducible ideals are primary in a Noetherian ring but not necessarily otherwise.
p. 17 line -10.	In a defining family for a Krull domain R, not every V_i must be of the form R_P; however, if R is a Krull domain, then the set of all R_P for prime ideals of height 1 is a defining family.
p. 24 lines 9-10.	In defining INC, we need to assume that $P \subseteq Q$ as well as $P \cap R = Q \cap R$.
p. 25 lines 19-20.	Given a prime P of R, infinitely many primes of $R[X]$ lie over P, namely $PR[[X]]$ and (P, f), where f is a polynomial irreducible modulo P.
p. 31 lines 14-15.	$PR[[X]]$ is not necessarily a prime ideal in $R[[X]]$, but $P[[X]]$ is prime; $P[[X]]$ consists of all power series having all coefficients in P. $P[[X]]$ is usually much larger than $PR[[X]]$.
p. 33 line 10.	If R is Noetherian and I is contained in the Jacobson radical of R, then for any ideal J of R, we have $\bar{J} \cap R = J$.
p. 37 lines 7-8.	In the definition of basis of a free module, we need not only that $a_iR \cap a_jR = \{0\}$ for $i \neq j$, but also that $a_iR \cap (\sum_{j \neq i} a_jR) = \{0\}$ for $i \neq j$.
p. 46 lines 8-9.	If $R[X]$ has a maximal ideal M with $M \cap R = \{0\}$, then certainly $K \subseteq R[X]/M$, but they need not be equal.
p. 48 line -1.	If R is VNR, then every finitely generated ideal is projective, but not necessarily every finitely generated module. See [6, Chapter 4].
Ex. 1 (f) line 1.	R is Hilbert if and only if K is uncountable. The easiest way to prove this is to use the fact that in a countably generated algebra over an uncountable field, the Jacobson radical is a nil ideal, i.e., every element is nilpotent.
Ex. 37 (e) line -2.	Delete the parenthetical remark.
Ex. 76 (c)-(d).	If $m - 1$ or $m + 1$ is divisible by 9, then either $R = \mathbb{Z}[s, t]$ or $R = \mathbb{Z}[s]$. If $m = 10$, then $R = \mathbb{Z}[s]$, and if $m = 19$, then $R = \mathbb{Z}[s, t]$.

(f).	Notice that other cases such as $m = 20$, or in general $m = ab^2$, have not been covered.
Ex. 93.	The existence of the element a cannot be taken for granted; there are integrally closed domains which are not completely integrally closed in which no such element can be found. If R is a 2-dimensional valuation domain, however, we definitely can find such an element. See [5].
Ex. 162 (d)-(e).	The generators of the free abelian groups in question are the monic irreducible polynomials in X (or in X and Y, respectively) with non-zero constant terms.
Ex. 172 (a) line 1.	R is not quasi-local.
Ex. 174 (c) line 1.	T is not a Hilbert ring. This corresponds to an error in [3, Theorem 3.3].
Ex. 177 (b) line 3.	$MD[[X]]$ is not necessarily a prime ideal. In [1], it was shown that if a domain R possesses a prime ideal P with $P[[X]] \neq \operatorname{rad}(PR[[X]])$, then $\dim(R[[X]]) = \infty$.
(f) line 3.	If D_P is a Noetherian valuation domain and $P^* = PD[[X]]$, then $D[[X]]_{P^*}$ is also a valuation domain.

References

[1] Jimmy T. Arnold, *Krull dimension in power series rings*, Transactions of the American Mathematical Society **177** (1973), 299–304.

[2] ———, *Power series rings over discrete valuation rings*, Pacific Journal of Mathematics **93** (1981), no. 1, 31–33.

[3] Robert Gilmer, *On polynomial rings over a Hilbert ring*, The Michigan Mathematical Journal **18** (1971), no. 3, 205–212.

[4] William Heinzer and Jack Ohm, *An essential ring which is not a v-multiplication ring*, Canadian Journal of Mathematics **25** (1973), 856–861.

[5] Jack Ohm, *Some counterexamples related to integral closure in D[[x]]*, Transactions of the American Mathematical Society **122** (1966), no. 2, 321–333.

[6] Joseph J. Rotman, *Notes on homological algebra*, vol. 26, Van Nostrand Reinhold, 1970.